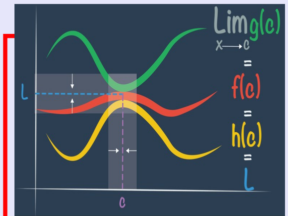
Math 261

Fall 2022

Lecture 5



Evaluate

1) $\lim\limits_{x \to 4} \dfrac{x^2-4x}{x^2-3x-4} = \dfrac{4^2-4(4)}{4^2-3(4)-4} = \dfrac{0}{0}$ I.F.

$\lim\limits_{x \to 4} \dfrac{x^2-4x}{x^2-3x-4} = \lim\limits_{x \to 4} \dfrac{x\cancel{(x-4)}}{\cancel{(x-4)}(x+1)} = \lim\limits_{x \to 4} \dfrac{x}{x+1} = \boxed{\dfrac{4}{5}}$

1) $\dfrac{1}{5}$

2) $\lim\limits_{t \to 1} \dfrac{t^4-1}{t^3-1} = \dfrac{1^4-1}{1^3-1} = \dfrac{0}{0}$ I.F. $\to A^3-B^3 = (A-B)(A^2+AB+B^2)$

$\lim\limits_{t \to 1} \dfrac{(t^2+1)(t^2-1)}{(t-1)(t^2+t+1)} = \lim\limits_{t \to 1} \dfrac{(t^2+1)(t+1)\cancel{(t-1)}}{\cancel{(t-1)}(t^2+t+1)}$

$= \lim\limits_{t \to 1} \dfrac{(t^2+1)(t+1)}{t^2+t+1} = \dfrac{2 \cdot 2}{3} = \boxed{\dfrac{4}{3}}$ 2) $\dfrac{4}{3}$

3) $\lim_{x\to -4} \frac{\sqrt{x^2+9}-5}{x+4} = \frac{0}{0}$ I.F. A^2-B^2

$$\lim_{x\to -4} \frac{\sqrt{x^2+9}-5}{x+4} = \lim_{x\to -4} \frac{\overbrace{(\sqrt{x^2+9}}^{A}\overbrace{-5)}^{-B}\overbrace{(\sqrt{x^2+9}}^{A}\overbrace{+5)}^{+B}}{(x+4)(\sqrt{x^2+9}+5)}$$

$$= \lim_{x\to -4} \frac{(\sqrt{x^2+9})^2-(5)^2}{(x+4)(\sqrt{x^2+9}+5)} = \lim_{x\to -4} \frac{x^2+9-25}{(x+4)(\sqrt{x^2+9}+5)}$$

$$= \lim_{x\to -4} \frac{x^2-16}{(x+4)(\sqrt{x^2+9}+5)} = \lim_{x\to -4} \frac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(\sqrt{x^2+9}+5)}$$

$$= \lim_{x\to -4} \frac{x-4}{\sqrt{x^2+9}+5} = \frac{-4-4}{\sqrt{(-4)^2+9}+5} = \frac{-8}{10}$$

$$= \boxed{\frac{-4}{5}} = \boxed{-.8}$$

$$\lim_{h\to 0} \frac{(3+h)^{-1}-3^{-1}}{h} = \frac{(3+0)^{-1}-3^{-1}}{0} = \frac{3^{-1}-3^{-1}}{0} = \frac{0}{0} \text{ I.F.}$$

$$\lim_{h\to 0} \frac{(3+h)^{-1}-3^{-1}}{h} = \lim_{h\to 0} \frac{\frac{1}{3+h}-\frac{1}{3}}{h}$$

$LCD = 3(3+h)$

$x^{-n} = \frac{1}{x^n}$

$$= \lim_{h\to 0} \frac{3-(3+h)}{h\cdot 3(3+h)} = \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}\cdot 3(3+h)}$$

$$= \lim_{h\to 0} \frac{-1}{3(3+h)} = \boxed{\frac{-1}{9}}$$

Is there a number a such that

$$\lim_{x \to -2} \frac{3x^2 + ax + a + 3}{x^2 + x - 2}$$ exist? if so

find a & limit value.

$$\lim_{x \to -2} \frac{3x^2 + 15x + 18}{(x+2)(x-1)}$$

Plug in -2 in the numerator and we should get 0.

$3x^2 + ax + a + 3$

$3(-2)^2 + a(-2) + a + 3 = 0$

$12 - 2a + a + 3 = 0$

$15 - a = 0$

$\boxed{a = 15}$

Now $$\lim_{x \to -2} \frac{3x^2 + 15x + 18}{x^2 + x - 2} = \lim_{x \to -2} \frac{3\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x-1)}$$

$$= \frac{3(-2+3)}{-2-1} = \boxed{-1}$$

$f(x) = \dfrac{x^2 + x - 6}{|x-2|}$ ← $x \neq 2$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{x^2 + x - 6}{|x-2|} = \frac{2^2 + 2 - 6}{|2-2|} = \frac{0}{0} \text{ I.F.}$$

Recall

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

what about $|x-2|$

$$|x-2| = \begin{cases} -(x-2) & \text{if } x < 2 \\ x-2 & \text{if } x \geq 2 \end{cases}$$

$x > 2$

$|x-2| = -(x-2)$ $|x-2| = x-2$

$\longrightarrow 2 \longleftarrow$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \frac{x^2 + x - 6}{x-2} = \lim_{x \to 2^+} \frac{(x-2)(x+3)}{x-2} = \boxed{5}$$

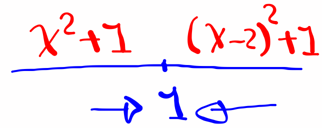
$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{x^2 + x - 6}{-(x-2)} = \lim_{x \to 2^-} \frac{(x-2)(x+3)}{-(x-2)} = \boxed{-5}$$

Find $\lim_{x \to 2} f(x)$ D.N.E.

why?

$\lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x)$

$$f(x) = \begin{cases} x^2+1 & \text{if } x<1 \\ (x-2)^2+1 & \text{if } x \geq 1 \end{cases}$$

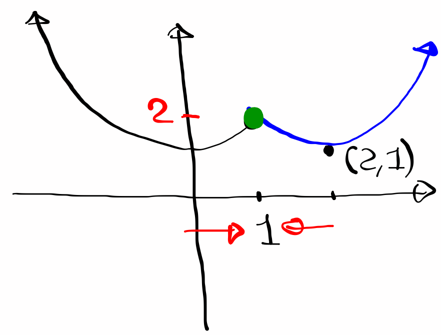


$\lim_{x \to 1^-} f(x) = 1^2+1 = \boxed{2}$

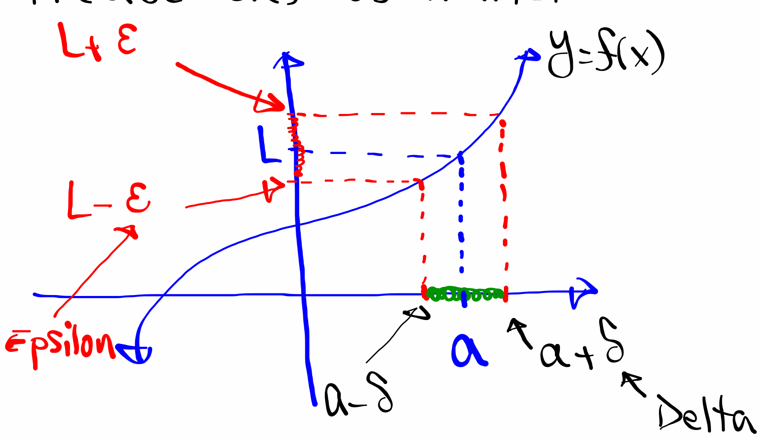
$\lim_{x \to 1^+} f(x) = (1-2)^2+1 = \boxed{2}$

$\lim_{x \to 1} f(x) = 2$

Since $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 2$



Precise def. of limits:



For any $\varepsilon > 0$, there is a $\delta > 0$ Such that

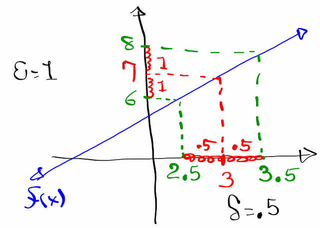
if $|x-a| < \delta$, then $|f(x)-L| < \varepsilon$

$\lim_{x\to 3}(2x+1)=7$

$f(x)=2x+1$

$a=3$

$L=7$



$f(x)=6 \quad 2x+1=6 \quad x=5/2$

$f(x)=8 \quad 2x+1=8 \quad x=7/2$

$|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$|2x+1-7|<\varepsilon$ whenever $|x-3|<\delta$

$|2x-6|<\varepsilon$ whenever $|x-3|<\delta$

$|2(x-3)|<\varepsilon$ whenever $|x-3|<\delta$

$|AB|=|A||B|$

$2|x-3|<\varepsilon$ whenever $|x-3|<\delta$

$|x-3|<\frac{\varepsilon}{2}$ whenever $|x-3|<\delta$

Pick $\delta=\frac{\varepsilon}{2}$